ISI – Bangalore Center – B Math - Physics I –Back paper Exam
Date: 7 June 2017. Duration of Exam: 3 hours
Total marks: 90

Answer ALL questions

Q1: Total marks: 6+6=12

a.) An insect moves on a spiral trajectory in a plane such that its polar coordinates at time t are given
by $r = be^{\Omega t}, \theta = \Omega t$, where b, Ω are positive constants.

Find the velocity and acceleration vectors of the insect at time *t,* and show that the angle between these vectors is always $\pi / 4$.

b.) A particle P moves so that its position vector $\vec{r}$ satisfies the differential equation
$\dot{\vec{r}} = \vec{c}$ x $\vec{r}$.
where $\vec{c}$ is a constant vector. Show that P moves with constant speed on a circular path.

Q2: Total marks: 4+4+4=12

An overdamped harmonic oscillator satisfies the equation $\ddot{x} + 10\dot{x} + 16x = 0$.

a.) Show that the most general solution to this equation is given by $Ae^{-2t} + Be^{-8t}$ where *A,B* are arbitrary constants.

b.) At time *t*=0, the particle is projected from the point *x*=1 towards the origin with speed *u*. Find *x*(t) in the subsequent motion.

c.) Show that the particle will reach the origin at some later time *t* given by

$$e^{6t} = \frac{u-2}{u-8} .$$

How large must *u* be so that the particle will pass through the origin?

Q 3 [Total Marks: 6+6+4=16]

A mass *m* whirls around on a string on a frictionless table. The string passes through a hole at the centre of the table and is pulled down to maintain tension in the string. Neglect gravity. Initially the mass is at a distance r_0 from the centre and is revolving at constant angular velocity ω_0. Starting at $t = 0$, the string is pulled with constant speed V so that the radial distance of the mass to the centre decreases.

a.) Show that the angular velocity is given by $\omega(t) = \omega_0 \frac{r_0^2}{(r_0 - Vt)^2}$

b.) Show that the tension is the string is given by $T = mr_0\omega_0^2\left(\frac{r_0}{r_0 - Vt}\right)^\alpha$ and determine the value of α.

c.) What is the physical reason that the tension is increasing as the string is pulled in?

Q 4. [Total Marks: 6+6+6=18]

a.) Suppose a general system of particles has a total mass M and in one particular frame of reference its Centre of Mass (CM) has velocity $\vec{V}$. Show that the kinetic energy *T* as calculated is this frame is related to T^{CM} (where T^{CM} is the kinetic energy calculated in the frame where the CM is at rest) by the following equation:

$$T = T^{CM} + \frac{1}{2}M\,|\vec{V}|^2$$

b.) Using the result in part a.) show that loss or gain of kinetic energy in a collision process is the same regardless of the inertial frame of reference in which the loss or gain is calculated.

c.) Two particles with masses m_1, m_2 and velocities $\vec{v}_1, \vec{v}_2$ collide and stick together. Find the velocity of this composite particle and show that the loss in kinetic energy due to the collision is $\frac{m_1 m_2}{2(m_1 + m_2)}|\vec{v}_1 - \vec{v}_2|^2$

Q 5. [Total Marks: 4+12 +4=20]

a.) Show that if the component of the total external torque along a fixed direction is zero, then the component of the total angular momentum along that direction is conserved.

b.) A fairground target consists of a uniform circular disk of mass M and radius a that can turn freely about a diameter which is fixed in a vertical position. Initially the target is at rest. A bullet of mass m is moving with speed u along a horizontal straight line at right angles to the target. The bullet embeds itself in the target at a point which is at a distance b from the fixed diameter.

Show that the component of angular momentum along the fixed vertical diameter is conserved.

Show that the angular speed of the target after the collision is given by

$$\frac{4mbu}{Ma^2 + 4mb^2}$$

[The moment of inertia of the disk about its rotation axis is $\frac{1}{4}Ma^2$].

c.) Determine if the collision is elastic.

Q 6. [Total Marks: 4+4+2+4=12]

a.) Two particles of equal masses are confined to move along the x-axis and are connected by a spring of length l which stores potential energy $V = \frac{1}{2}k(\Delta x)^2$ where (Δx) is the expansion or compression of the spring from its natural length l.

Write down the Lagrangian L in terms of the coordinates x_1, x_2 of the particles with respect to a fixed origin.

b.) Rewrite L in terms of the new variables $X = \frac{1}{2}(x_1 + x_2)$ and $x = x_1 - x_2 - l$, and write Lagrange equations for X and x.

(c) Solve for X(t) and x(t) and describe the motion.

d.) Describe the conserved quantities in this system.